OPEN CALCULATION METHOD BASED ON NUMBERS (ABN)

A new way to teach mathematical calculation

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INTRODUCTION

All teachers teach the same mathematical calculation method and everyone learns the same procedure. However, adults most often need to solve mathematical problems without paper and pencil and do not apply the instructions they have learned. In order to do this we mentally calculate with methods quite different to the way they have been traditionally taught. For this reason, another way to teach mathematical calculation is necessary.

Teachers and educational researchers are conscious that the closed traditional method based on ciphers (CBC) presents serious problems and difficulties in calculation and solving problems because it is an irrational method, especially for the students with difficulties. These disadvantages of the traditional method have become a favourite topic for analysis in a great amount of issues, such as Ablewhite (Ablewhite, 1971) and other, more recent, authors (Ferrero, 1984; Alcalá, 1986; Kamii, 1986; Castro, Rico y Castro, 1987; Baroody, 1988; Maza, 1989; Resnick y Ford, 1990; Dickson, Brown y Gibson, 1991; Vergnaud, 1991 Gómez Alfonso, 1999; Chamorro, M.C. (coord.), 2005).

The main differences between the CBC and the open calculation method based on numbers (ABN) are that the former is a closed method because only one answer is possible: children apply the same consideration to each cipher separately and the process is the same for each amount. The new method ABN, however, is open and personalized because students can solve operations in multiple ways, each student solves operations easily, comprehensibly and without stress using a table where they compose and decompose numbers freely. In this method, students acquire a real number and amount concept and each child develops the necessary steps, thus it is adaptable to each individual.

The purpose of this text is to expose the need for a methodological change in our teachers and classrooms because a new method to teach mathematical calculation is possible. Furthermore, in addition to expounding the new mathematical learning method (ABN), the text also will highlight the importance of diversity awareness in mathematics. Additionally, an explanation about both methods will be given.

1. METHODOLOGICAL CHANGE

In this section, the main reasons for this change are presented because the traditional method (CBC) has several faults and problems for children. The CBC method has been used for many years and it was created in a quite different context. The origins of this method had an economic aim, for the economy and transactions in the markets, but no an academic goal.

A great deal of research has demonstrated that with this method (CBC):

- Pupils do not learn to calculate
- Pupils do not learn to solve problems
- They present a negative attitude towards mathematical learning
- Pupils have more possibilities to fail than to pass.
- This situation has been going on for tens of years.

The persistence of poor performance over time indicates that the problem is deeply-rooted. This problem affects the current whole paradigm of mathematical learning, therefore the greatest trouble is the calculation system used by teachers in all schools. Later, when the purpose of math teaching using the traditional method is explained, the main reason of this problem will be specified.

2. DIVERSITY AWARENESS IN MATHEMATICS

According to Martínez Montero, J. (2011) in the document about *"Diversity Awareness in Mathematics: A Methodological and curricular approach"*, most of the proposals which were with respect to the awareness of children's different capacities and skills, are focused on organizative or procedimental aspects and not on curricular aspects. This situation is common in all curricular areas, above all in mathematics. The different points of view offered to the children are concentrated on three aspects:

– The learning moment (learning is slowed down when children need special attention).

– Learning speed (the contents are worked on more slowly).

– Learning amount (some learning contents are eliminated for the children with difficulties).

Regarding gifted pupils no special treatment exists. This diversity awareness is produced for an obsolete method, based on mechanical learning of calculation and behaviourist psychological approach. The central part of the traditional mathematical learning consists of times tables memorization, the basic combination of digits and the instruction set which is followed to solve the operations (addition, subtraction, multiplication and division). For example, the multiplication operation, 3247 x 7, is solved following the steps below:

1. Memorization times tables of the number 7 and remembering the numbers.

2. Division of the multiplicand in ciphers (3-2-4-6).

3. The child has to take the cipher of the right (6), and it is matched with the number 7. Now, the result is searched for in the memory (6 x 7=42).

4. When the number is found, and it is smaller than 10, it below the line and the first cipher on the right. If this number is bigger than 10, only the cipher of the units (2) is written and the child has to keep in his/her memory the cipher of the tens (4).

5. Now, the next cipher on the left is taken (4). It is matched with the number 7. The result is searched for in the memory (7 x 4 = 28).

6. The number kept in the memory is added to this number (28 + 4 = 32). This number is written to the left of the last number below the line (2) the cipher of the units (2) and another cipher of tens is kept in the memory (3).

7. The same process is done with the other ciphers (2 and 3).

8. With the last number (3), the product is written in full (3 x 7 = 21).

9. Finally, the value obtained is read.

This learning to solve operations is a boring and memoristic method, because only two skills are necessary: memorizing the times tables and knowing how to add the digit kept in the memory to a number with two ciphers. The process is difficult because the pupil does not understand anything and at no time knows what she/he does.

Attending to diversity from a methodological and curricular approach in mathematics involves:

I. Utilizing learning processes in accordance with the children's maturity and development.

II. Increasing flexibility of the algorithmic options offered to pupils to adapt them to the realities which represent and symbolize.

III. Facilitating multiple ways to solve operations depending on the different capacities and skills of each child.

3. CLOSED CALCULATION METHOD BASED ON CIPHERS (CBC TRADITIONAL METHOD)

CBC is a closed method because only one answer is possible: children apply the same consideration to each cipher separately and the process is the same for each amount. This method produces several difficulties and faults in the mathematical learning. The main problem of this method is its purpose, the numerical model which it is based on and the need to follow inflexible and illogical procedures and rules to solve the operations.

3.1. The purpose of math teaching with the traditional method.

The current mathematical methodology was created several years ago. The purpose of this method was ensuring that children solved operations in a fast and accurate way, because there were no machines to assist in calculation. Mathematical learning was not used to educate pupils, but rather the children's potentiality and skills were used to obtain a calculation tool.

The purpose of mathematical learning was and is that pupils do addition, subtraction, multiplication and division operations in a fast and correct way, and not that they learn to add, subtract, multiply or divide. All mathematical and methodological aspects were sacrificed. Nowadays, this purpose is meaningless and it is explained for the reasons below:

– Currently, a great number of calculation machines exist and these machines calculate better and faster than us.

– Anyone who is an adult calculates without pencil and paper, because this method is only useful at the beginning of mathematical learning.

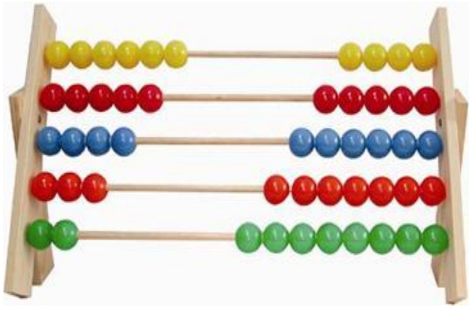
– To teach mental calculation is the best way to teach mathematical calculation.

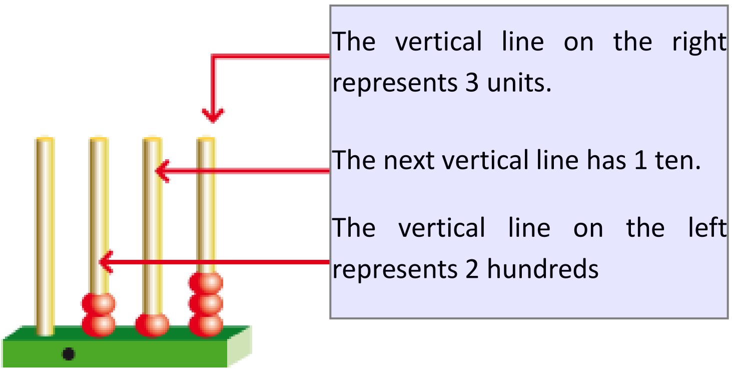
The aim of mathematical learning should be to prepare the children to acquire, understand and apply the knowledge and the mathematical tools in their everyday life. In brief, teachers should train pupils to be competent in mathematics. The objective will not be to do mechanical calculation, but to use the training potentialities of calculation (and mathematics in general) to enhance the children's intellectual development and to increase their mathematical competency.

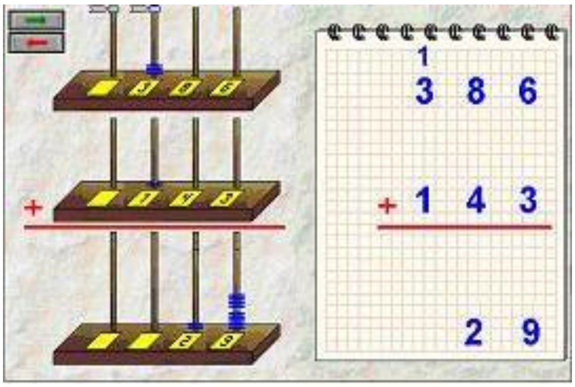
3.2. Numerical model used (Abacus)

The most common numerical model used is based on the abacus. Around the world, the abacus has been used in pre-schools and elementary schools as an aid in teaching the numeral system and arithmetic.

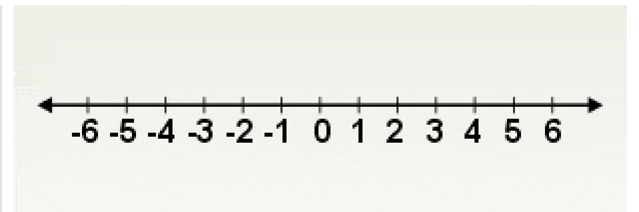
In Western countries, a bead frame is used with straight wires and a vertical frame has been common. It is still often seen as a plastic or wooden toy. The illustrations below present the most typical abacus used in schools:

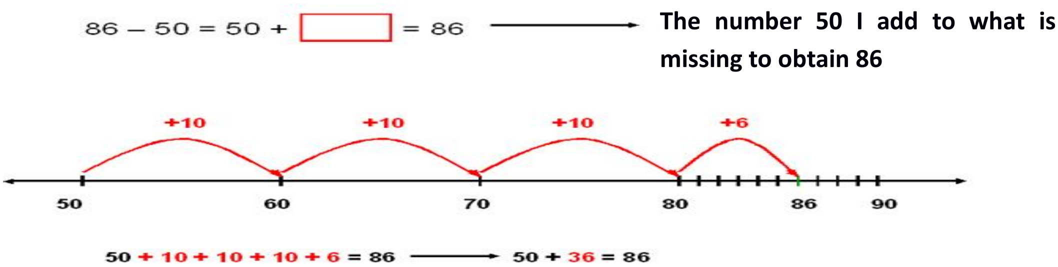






However, the numerical model should be more flexible, logical and visual. The next illustrations represent some examples of this numerical model:





0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

It is written	One hundred	One hundred one	One hundred two	One hundred three	One hundred four	One hundred five	One hundred six	One hundred seven	One hundred eight	One hundred nine
It is read	100	101	102	103	104	105	106	107	108	109
	110	111	112	113	114	115	116	117	118	119
	120	121	122	123	124	125	126	127	128	129
	130	131	132	133	134	135	136	137	138	139
	140	141	142	143	144	145	146	147	148	149
	150	151	152	153	154	155	156	157	158	159
	160	161	162	163	164	165	166	167	168	169
	170	171	172	173	174	175	176	1767	178	179
	180	181	182	183	184	185	186	187	188	189
	190	191	192	193	194	195	196	197	198	199

The numerical model through the abacus has serious problems:

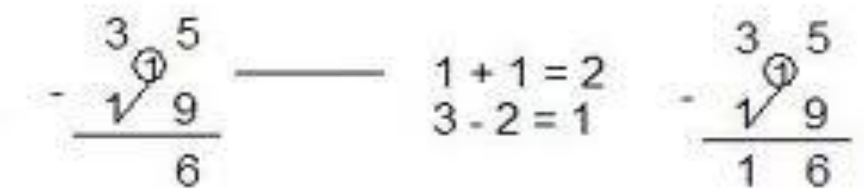
- It represents a false vision of the number and numerosity. For instance, to count 25 with the abacus becomes a difficult and illogical process. The main question is how a child could count more easily and comprehensibly 25, using the abacus or the numerical table:

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

– All operations are solved as if the numbers represent amounts lower than ten. Children apply the same treatment to each cipher separately

– It requires an inflexible calculation procedure and only one answer is possible.

The next example shows a subtraction where the numbers receive the same treatment regardless of their position and amount represented, all ciphers are solved as units. For this reason, this model is irrational and illogical. Consequently, children do not link the number and amount concept.



This way of calculation prevents achieving real operations because the numbers are not units of significance, they are obtained conversely to how the brain processes. In addition the composition and decomposition of numbers is performed by placing a number next to another. In consequence, it prevents the estimation in the calculation operations, both mental and written.

3.3. Problems with the numerical model in the operations.

<u>Subtractions</u>

Subtraction is the operation which produces most problems. When the number on the top is smaller than the number on the bottom, the difficulties and the absurd process begin. The two biggest problems with this kind of subtractions are:

1. The procedure is absurd; therefore children do not understand the process and solve operations following the rules and a mechanical way.

2. The children perform an unreal and different operation.

As an illustration, the operation proposed and that performed are presented below.

PROPOSED OPERATION

7	0	0
- 1	5	6
5	4	4

PERFORMED OPERATION

8	1	0
- 2	6	6
5	4	4

Multiplications

The problems presented by the multiplication operations are:

- Opacity. The procedure is inflexible as in the other operations and do not represent a real numbers and amounts.

		6	3	2
		X	8	9
	5	6	8	8
5	0	5	6	
5	6	2	4	8

CLOSED CALCULATION METHOD

(CBC Method)

X	600	30	2	Total
80	48000	2400	160	50560
9	5400	270	18	5688
Total	53400	2670	178	56248

OPEN CALCULATION METHOD

(ABN Method)

- The zeros are interspersed.
- The impossibility of fragmentation.
- The inability to practice all multiplicative structures.

Division

The main problems with the division operations are:

– The problem of the order of magnitude.

– The problem of zero in the quotient.

– The problem of the decimal numbers in the divisor.

4. OPEN CALCULATION METHOD BASED ON NUMBERS (ABN METHOD)

The alternative to the traditional method (CBC) has to follow the following principles:

- Calculation based on numbers.

- Numbers considered from different points of view.

- Open calculation method.

- Transparent calculation.

-Realistic estimate, referenced.

- Calculation with real problem solving.

- Calculation with estimation and approximation.

4.1. History and characteristics of the ABN method.

The new methodology was created by Jaime Martinez Montero. This methodology began from his Doctoral Thesis, published in 1995, in which the author proposed a complementary approach to the operations of learning and examined many of the students' difficulties in problem solving, and where these difficulties originated. Another important contribution to this new methodology was the first article by the author written in 2001, “Unwanted effects (and devastating) of traditional learning methods of numbering and the four basic operations algorithms”.

The first book (Martinez-Montero, 2000) explains an alternative proposal to mathematical operations, whereas in his second book (Martinez Montero, 2008) he details the new system of calculus. Finally, the last book incorporates this new approach in the remedial teaching of mathematics (Martinez Montero, 2010). The ABN is a method for mental arithmetic and problems solving and is different because it promotes real learning. It’s called ‘open’ because the students can solve operations and problems in several ways, being a method which caters for the individual progress of each student. Nowadays, more books have been published (Martinez Montero, 2011) about this methodology which are used by teachers for teaching in the classroom.

The main objective of this new method is eliminating the traditional formats of the basic operations and replacing them for open calculation based on a numbers format. The adoption of this new methodology attempts to achieve the total change in the teaching-learning process of calculation and problem solving in primary education, using as formal support for problem learning a model based on semantic categories. Next years will be applied in secondary education. Secondary objectives, but no less important are:

1. Improving mental calculation and estimation capability.
2. A significant improvement in the ability to solve problems.
3. The creation of a favourable attitude to learning mathematics.

The open calculation method based on numbers (ABN) is open and personalized because students can solve operations in several ways, each student solves operations comfortably, easily and comprehensibly using a table, where they compose and decompose numbers freely, acquiring a real number and amount concept and each child develops the necessary steps thus adapting the method to each individual.

4.2. Advantages.

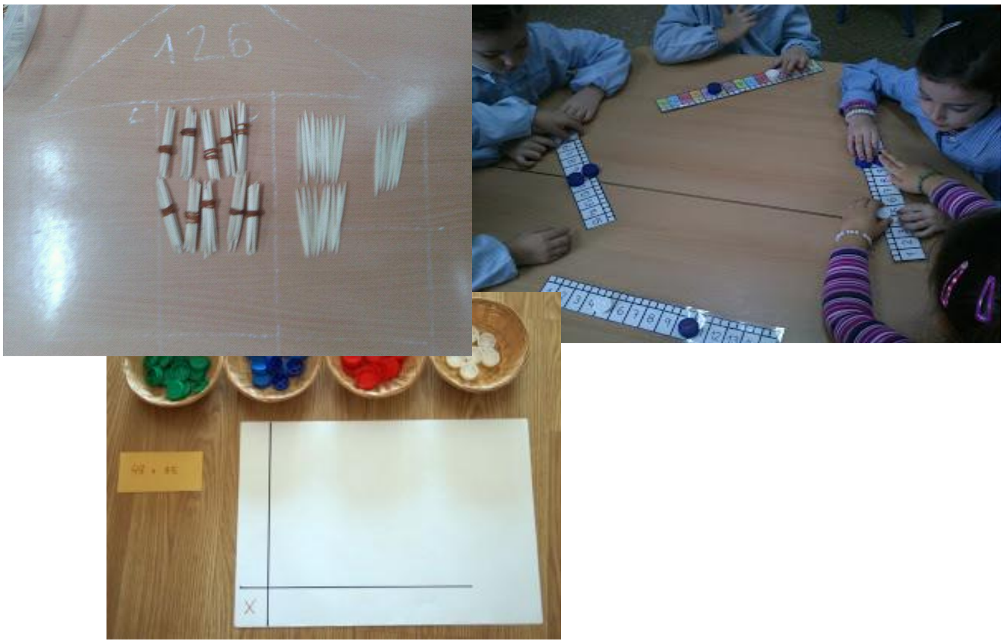
The open calculation based on numbers method (from henceforth, ABN) has two types of advantages. The first is that it makes many of the difficulties of traditional algorithms disappear, such as illogical subtraction, no zeros to the quotient... The second one is that it allows each student to use his/her own processing system in the calculations; for this reason it is called open calculation method.

ABN algorithms are based on numbers, which facilitates the relation to the children's natural intuitive processes and develops a dynamic approach of the number sense. Additionally, these algorithms are open and adjustable, transparent, are developed with the use of references, facilitate the story of what is done and finally develop a higher capacity for estimating. For this reason, this method could prevent the difficulties students presently have and solve them if they occur.

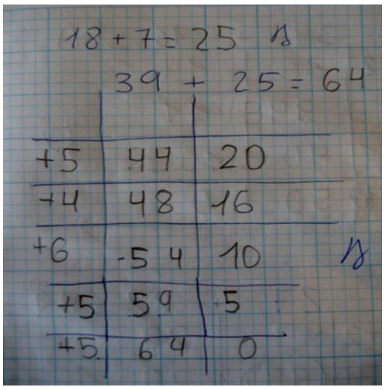
4.3. Procedure for the different operations:

- **ADDITION**

The essence of addition is to accumulate an addend to the other. Once fully accumulated, the new addend will give us the result. In the traditional algorithm, the format can be only done one way: decomposing the addends into units, tens, hundreds,...; placing them properly and, finally, making a combination unit to unit and following the order from lesser to higher (no exceptions and no possibility to modify this rule are allowed). At the beginning of the learning, children use different manipulative materials such as toothpicks, lids…

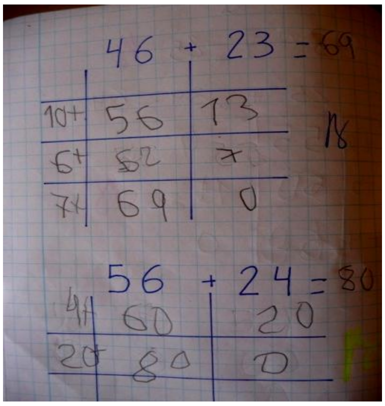


Addition has different phases:



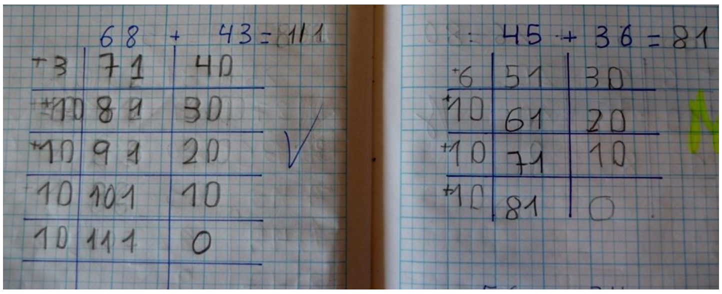
→ *__1st phase:__*

The child adds 25 to 39, but does not see the addends as two tens and five units, but treats them as if all its members were drives; that is, as if adds toothpick to toothpick.



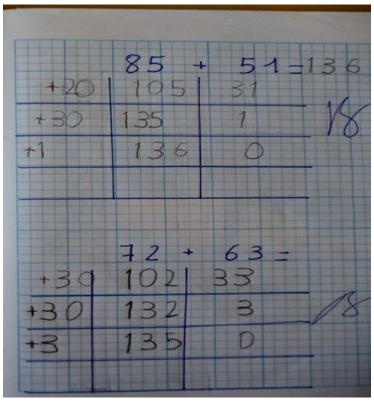
→ *__Transition phase:__*

In this stage an evolution is presented clearly. Children begin to solve the addition into two actions: units and tens in one go and separately.



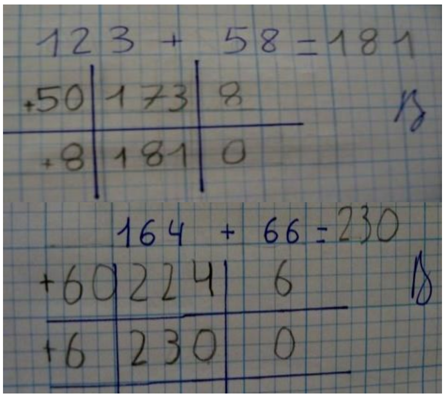
→ *2nd phase:*

In this phase, for some children the numbers represent a set of tens and loose units.



→ *Transition to 3rd Phase. Grouping of tens:*

In this case, the number of tens is divided to operate better, but the set of tens is higher than in the last phase.

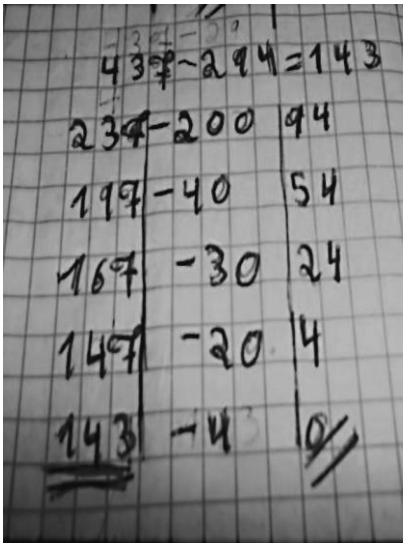


→ 3*rd Phase:*

The last stage is more complex and children can solve the operations in different ways, presenting a higher capability or domain to combine tens and units.

- **SUBTRACTION**

In subtraction three different basic models are used, which are adapted to different types of problems. The first is used to detraction and comparison problems (problemas de detracción y comparación) as set out below (Martinez Montero, 2008):



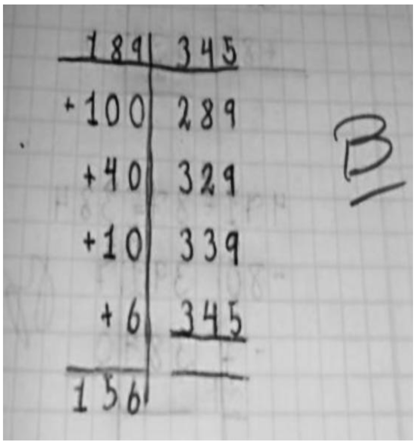
→ *Detraction and comparison format*

This type of subtraction is solved by removing from both terms the same amount until the smallest disappears. The resulting number of the highest amount is the result.

In this problem type (i.e. I had 437€ and I spent 294€. How many euros do I have?) the first column indicates the initial amount, the second one the amount which I am spending and the third one the amount which I can spend. In some schools the column positions can vary depending on the teachers.

→ *Ascending stair format*

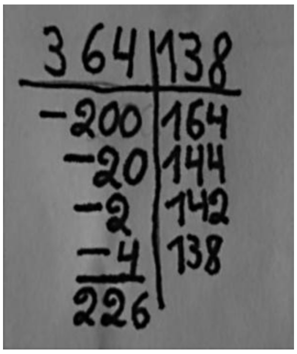
This subtraction process is the most natural and the most likely to be employed by children. It is the system used in the operations of giving change in shops. This format only uses two columns. In the first one, the partial amounts are collocated. In the second one, the progress is represented until the desired amount is obtained.



The process is easy to explain. Firstly, from the subtrahend, the amounts are added until to arriving to the minuend. Adding to the subtrahend (first column) the necessary amounts, and in the second column the obtained amounts are written. When the number requested is obtained, add all the amounts to obtain the result. In this case, the addition is more worked than the subtraction.

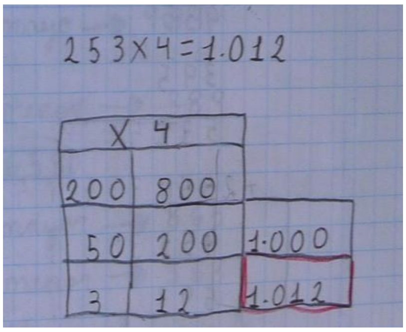
→ *Descending stair format*

The process followed in this model is opposite to the ascending stair format. In this case, the highest amount is converted in the smallest amount, in two columns. The typical problem model for this format is: How many floors do I have to descend from the 364th floor to the 138th? The example shows the operation realized by a child in second grade and the steps to solve this problem.

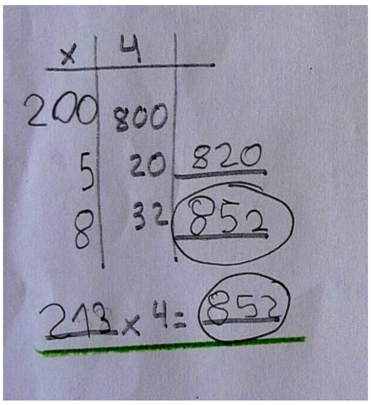


- **MULTIPLICATION**

In the case of multiplication, there are fewer alternatives because is an operation which requires the memorization of times tables. The process is simple, decomposing the factor in units, tens, hundreds, and then the partial products are added.



The example below shows how some children decompose the numbers in a different way depending on the children's capacity. This example shows the multiplication, 213x4. Firstly, the child decomposed the hundreds and later decomposed the number 13 to 8 and 5, instead 10 and 3.

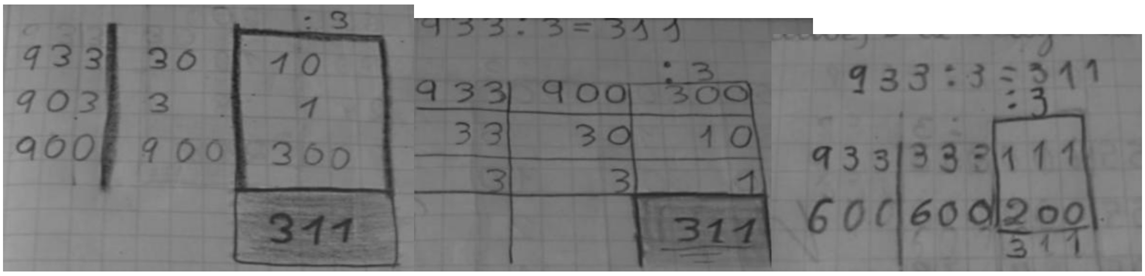


- **DIVISION**

Division allows different degrees of adaptation and breakdown of the calculations. In the initiation process, the children express different domains depending on their ability with the times tables. If a child controls the times tables, being able to multiply units, tens, hundreds and thousands, then he/she can solve all divisions by one cipher on the quotient.

This algorithm consists of three columns. The first one on the left represents the total amounts to be distributed. The second one, in the centre, represents the amounts taken by the child for doing the accurate distribution. The column on the right represents the partial quotients. The sum of them represents the total quotient and the amount in the first column is the remainder.

The next examples show the different ways to solve the divisions, depending on the children's skills.

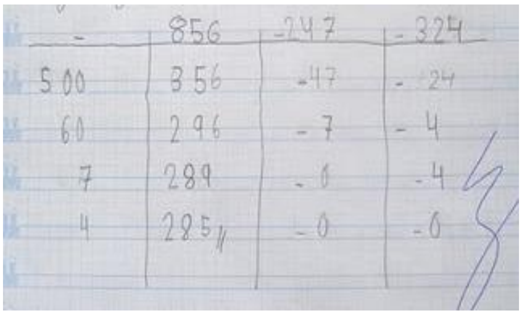


- **DOUBLE SUBTRACTION (DOBLE RESTA)**

This operation was created by a child in first grade who said to his teacher that if there are additions which have more than two addends why are there not subtractions with more than one subtrahend. And later, he tried it. This was correct because a subtraction with various subtrahends is possible in the ABN algorithm. This fact allows a problem to be converted from two operations into one, which is solved at once.

The example below shows a double subtraction realized by children with less calculation capacity, which present a conservative strategy. Firstly, subtract the first subtrahend (children who went on the excursion) and later subtract another subtrahend (those who went to the cinema).

There are 634 children at the school. 176 of them went on the excursion and 84 to the cinema. How many children are at the school?			
	684	-176	-84
130	554	46	84
54	500	46	30
70	430	6	0
6	424	0	0



When children have more ability and practice, they can use other strategies to solve the double subtraction. They use one or another subtrahend or both at the same time.

This last example represents the most advanced calculation level. The child groups two subtrahends into one and then subtracts it from the minuend at once.

There are 634 children at the school. 176 of them went on the excursion and 84 to the cinema. How many children are at the school?			
	684	-176	-84
+84	684	260	0
260	424	0	

- **ADD-SUBTRACT (SUMIRRESTA)**

This case is similar to previous operation because it also converts problems with two operations into another with one operation and facilitates more possibilities of calculation. This operation is used in combined operation problems which have to be solved through one subtraction and addition. In these operations, children have to indicate with signs if the operations are additions or subtractions

There are 634 children on the playground. 174 of them went on the excursion and later 105 came. How many children are there now?			
	634	-174	+105
-134	500	-40	+105
-40	460	0	+105
+105	565	0	0

At the first level, children solve one term and later solve the second one. In fact children solve two consecutive operations. In the example presented, first subtract the children who go on the excursion and then add those who arrive.

The second level is achieved when children combine both terms depending on the needs of calculation. This example on the left represents this level, in which both operations are solved at the same time, combining additions and subtractions.

There are 634 children on the playground. 174 of them went on the excursion and later 105 came. How many children are there now?			
	634	-174	+105
-134	500	-40	+105
+100	600	-40	+5
-40	560	0	+5
+5	565	0	0

The third level is produced when before confronting the final calculation; children reduce subtrahend and addend into a one term. With that term operates and leads to the result. This level involves the possession of appreciable calculation skills.

There are 634 children at the school. 176 of them went on the excursion and 84 to the cinema. How many children are at the school?			
	684	-176	-84
+84	684	260	0
260	424	0	

5. CONCLUSION

Research results support that mathematics could become a powerful tool for intellectual development of children, a fundamental piece in building the logical and critical thinking. Research findings demonstrate students through the ABN Method learn faster and better, because children's scores of calculation and estimation improve, as well as increase ability in solving problems in fact difficulties and mistakes of the traditional algorithm disappears.

This method is opening new ways to teach mathematical calculation and at this time, a great number of issues are beginning to evidence that it is a more motivational and logical method for children.

6. REFERENCES

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